Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2015

Name:______________________________________

Instructors: Garbett & Migliore

Exam I

February 5, 2015

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an $\times$ through your answer to each problem.

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Multiple Choice

1. (5 pts.) Let $U = \{1, 2, \ldots, 10\}$, $A = \{2, 4, 6\}$, $B =$\{odd numbers between 2 and 10\}. Which of the following statements is **false**?

(a) $n(B') = 6$ (b) $A \cup B = U$ (c) $A \cap B' = A$

(d) A and B are disjoint (e) $A \subseteq B'$

Solution:

We consider each answer choice.

(a) Notice that $B = \{3, 5, 7, 9\}$, so $B' = \{1, 2, 4, 6, 8, 10\}$. Thus, $n(B') = 6$, and the statement is true.

(b) We have $A \cup B = \{2, 3, 4, 5, 6, 7, 9\}$. Since this is not equal to U, this statement is false.

(c) From (a), we know $B' = \{1, 2, 4, 6, 8, 10\}$, so $A \cap B' = \{2, 4, 6\} = A$. This statement is true.

(d) Since B does not contain, $2, 4$, or 6, $A \cap B = \emptyset$, that is, A and B are disjoint. This statement is true.

(e) Since B' contains every element of A, $A \subseteq B'$, and this statement is true.

2. (5 pts.) Sammy's Pizzeria has 20 different pizza toppings to choose from. They currently have a deal where if you order a large pizza, you get up to 4 toppings free. How many pizzas that qualify for this deal can be made? [Note: A pizza with no toppings is also a possibility.]

(a) $C(20, 4)$

(b) $P(20, 4)$

(c) $P(20, 0) + P(20, 1) + P(20, 2) + P(20, 3) + P(20, 4)$

(d) $C(20, 0) \times C(20, 1) \times C(20, 2) \times C(20, 3) \times C(20, 4)$

(e) $C(20, 0) + C(20, 1) + C(20, 2) + C(20, 3) + C(20, 4)$

Solution:

A pizza that qualifies for the deal can have up to 4 toppings, so it can have exactly 0 toppings, 1 topping, 2 toppings, 3 toppings, or 4 toppings. However, it can only have one of these (a pizza can't have exactly 2 toppings and exactly 3 toppings), so we need to use the addition rule. The number of pizzas satisfying the deal is therefore

$$C(20, 0) + C(20, 1) + C(20, 2) + C(20, 3) + C(20, 4)$$

where $C(20, 0)$ is the number of pizzas with exactly 0 toppings (choose 0 of the 20 toppings), $C(20, 1)$ is the number of pizzas with exactly 1 topping (choose 1 of the 20 toppings), and so on. We use combinations because the order in which the toppings are chosen doesn't matter.

3. (5 pts.) Erin is competing in a race. She must first choose one of four obstacles. She must then kayak, canoe, or swim accross the river. Finally, she must either skateboard or inline skate to the finish. In how many ways can Erin complete the race?

(a) 2 (b) 24 (c) 9

(d) 84 (e) 4

Solution:
Erin must make 3 consecutive choices. First, she must choose an obstacle. There are 4 ways to do this. Having done this, she must choose a way to cross the river. There are 3 ways to do this. Having done this, she must choose a way to finish the race. There are 2 ways to do this. Since Erin is making consecutive choices, we can use the multiplication principle. There are $4 \cdot 3 \cdot 2 = 24$ ways for Erin to complete the race.

4. (5 pts.) How many 4 letter words (including nonsense words) can be made from the letters of the word

MACINTOSH

if letters **cannot** be repeated and the first letter must be a vowel?

(a) 1008 (b) 2058 (c) 3024

(d) 2673 (e) 1536

Solution:
Notice that the word MACINTOSH has 9 different letters where 3 of them are vowels. We make consecutive choices. First, choose the first letter. Since the first letter must be a vowel, there are 3 choices. Next, choose the second letter; there are 8 choices. Now, choose the third and then the fourth letters; there are 7 abnd 6 choices respectively. By the multiplication principle, the number of words is

$$3 \cdot 8 \cdot 7 \cdot 6 = 1008.$$

Alternately, you cold combine steps 2,3, and 4 into 1 step: choose letters 2,3, and 4. There are $P(8,3)$ ways to do this giving $3 \cdot P(8,3) = 1008$ words.

5. (5 pts.) Carrie is going on a trip. She has 20 books she would like to bring; 12 are mysteries and 8 are Science Fiction. In how many ways can Carrie choose 3 mysteries and 2 Science Fiction books to take on her trip?

(a) $P(20,5)$ (b) $C(20,5)$ (c) $P(12,3) \cdot P(8,2)$

(d) $C(12,3) \cdot C(8,2)$ (e) $C(12,3) + C(8,2)$

Solution:

We break this into two steps. Step 1 is to choose the 3 mysteries, and step 2 is to choose the 2 science fiction books. There are $C(12,3)$ ways to choose the 3 mysteries (we use combinations since the order in which the books are selected doesn't matter), and there are $C(8,2)$ ways to choose the 2 science fiction books. The multiplication principle gives

$$C(12,3) \cdot C(8,2)$$

ways to choose the books.

6. (5 pts.) The Finite Math Team and the Calculus Team are two Bookstore Basketball teams. Both have seven players, and they will play against each other in the first round. Both have to choose five members to start the game. How many possible choices are there for the five-against-five start of the game? In this problem we don't care which starters play which position. We just want to know who is starting.

(a) $P(7,5) \cdot P(7,5)$ (b) $(7!)(7!)$ (c) $\binom{7}{5} + \binom{7}{5}$

(d) $\binom{7}{5} \cdot \binom{7}{5}$ (e) $P(7,5) + P(7,5)$

Solution:

The number of ways for the Finite team to choose 5 starters is $\binom{7}{5}$. The number of ways for the Calculus team to choose 5 starters is also $\binom{7}{5}$. **For each** choice of the Finite starters there are $\binom{7}{5}$ starters for the Calculus team. Thus the correct answer is $\binom{7}{5} \cdot \binom{7}{5}$.

7. (5 pts.) Euchre is a card game played with only 24 cards, namely the ace, king, queen, jack, 10 and 9 of each of the four suits. Each player is given five cards. In how many ways can a player receive five cards that are all from the same suit? (Remember that there are four suits, six cards in each suit, and it doesn't matter in what order the player receives the cards.)

(a) $4 \cdot \binom{6}{5}$ (b) $4 \cdot P(6, 5)$ (c) $\binom{24}{5}$

(d) $\binom{24}{5} \cdot \binom{6}{5}$ (e) $6 \cdot \binom{24}{5}$

Solution:

In any given suit there are six cards, of which you are choosing five, so there are $\binom{6}{5}$ such choices. Since there are four suits, the final answer is $4 \cdot \binom{6}{5}$.

8. (5 pts.) The Philosophy Club of Notre Dame has 7 members. The annual Philosophers United convention will take place next month, and the ND club needs to send a delegation. The delegation can be any choice of **at least two** members of the club. How many possible delegations are there?

(a) 126 (b) 5040 (= 7!) (c) 120

(d) 125 (e) 128

Solution:

The total number of subsets of a set with 7 elements is $2^7 = 128$. In this case we rule out the set with no elements and the seven possible sets with only one element, leaving $128 - 1 - 7 = 120$ possibilities.

9. (5 pts.) An urn contains 13 numbered marbles, of which 7 are red and 6 are white. A sample of 5 is to be selected. How many possible samples are there where **at least** 4 are red?

(a) $\binom{7}{4} \cdot \binom{7}{5}$ (b) $\binom{7}{4} \cdot \binom{6}{1} + \binom{7}{5}$ (c) $\binom{7}{4} \cdot \binom{6}{1} \cdot \binom{7}{5}$

(d) $\binom{13}{4} + \binom{13}{5}$ (e) $\binom{7}{4} + \binom{7}{5}$

Solution:
The number of samples where exactly 4 are red and 1 is white is $\binom{7}{4} \cdot \binom{6}{1}$. The number of samples where all are red is $\binom{7}{5}$. Thus the correct answer is $\binom{7}{4} \cdot \binom{6}{1} + \binom{7}{5}$.

10. (5 pts.) A club has 15 members. In how many ways can they divide up into three (unordered) groups of five?

(a) $\frac{15!}{5!5!5!}$ (b) $\frac{1}{3!}\binom{15}{5}\binom{15}{5}\binom{15}{5}$ (c) $P(15,5)^3$

(d) $\binom{15}{5}\binom{15}{5}\binom{15}{5}$ (e) $\frac{1}{3!} \cdot \frac{15!}{5!5!5!}$

Solution:
Using the formula for unordered partitions, we get

$$\frac{1}{3!} \cdot \frac{15!}{5!5!5!}$$

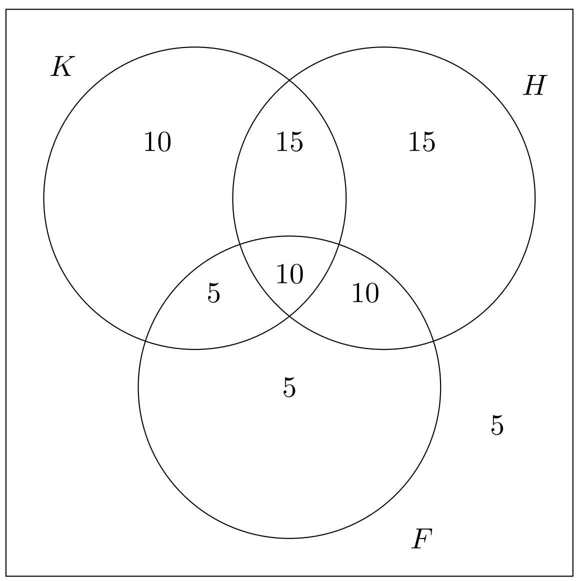
Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) The 75 members of the juggling club are given a survey. Here are the results.

- 10 club members like juggling fire, hammers, and knives
- 10 club members only like juggling knives and not the other two objects
- 15 club members are afraid of fire, so they don't like juggling fire, but do like juggling hammers and knives
- 20 club members like juggling hammers and fire
- 40 club members like jugging knives
- 50 club members like juggling hammers
- 30 club members like juggling fire

(a) If U ={club members}, K ={club members who like juggling knives}, H ={club members who like juggling hammers}, and F ={club members who like juggling fire}, fill in the Venn diagram with the results of the survey (you should have a number in every space).
Solution:



(b) How many club members (if any) do not like juggling any of the three objects?
Solution:
Five students don't like juggling any of the three objects.

12. (10 pts.) For this problem you do not need to simplify your answers; you may leave permutations, combinations, and factorials (i.e. $P(n,k)$, $C(n,k)$, $\binom{n}{k}$, or $n!$) or products or sums of numbers, permutations, combinations, or factorials in your answers if you so choose.

(a) The juggling club needs to elect officers for the coming year. The club wants to elect a president, a vice president, a secretary and a treasurer. Assuming any of the 75 members of the club can hold any position and no member can hold more than one position, how many ways are there to choose the four officers?

Solution:
We need to choose 4 of the 75 members to be officers. Order does matter because each of the 4 will have a different title. Thus, there are $P(75,4)$ ways to choose the officers.

(b) The juggling club also wants to have a club photo of all 75 members taken. The newly elected officers will stand in the front row, the second row will contain 21 members, and the third and fourth rows will contain 25 members each. How many different photographs can be taken? (Note: the four officers have already been chosen when the picture is taken, and these four have to be in the front row.)

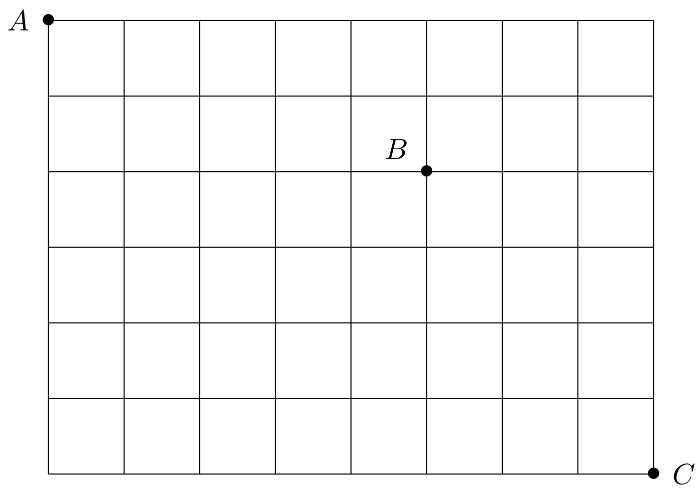
Solution:
We must split this into steps. First, we must choose the order in which the officers stand. There are 4! ways to do this. Next, choose who stands in the second row. There are $P(71,21)$ ways to do this (we can't choose the officers and order does matter). Now, choose who stands in the third row. There are $P(50,25)$ ways to do this (we can't choose people in the first or second row). Finally, choose who stands in the last row. There are $P(25,25)$ ways to do this. Using the multiplication principle, we find that

$$4! \cdot P(71,21) \cdot P(50,25) \cdot P(25,25)$$

different photos can be taken. Notice also that since there are no requirements for who stands in which row beyond the requirement that the officers must stand in the first row, once we've placed the officers, we can position everyone else in step 2 in 71! ways, giving us a total of $4! \cdot 75!$ photos.

13. (10 pts.) For this problem you do not need to simplify your answers; you may leave permutations, combinations, and factorials (i.e. $P(n,k)$, $C(n,k)$, $\binom{n}{k}$, or $n!$) or products or sums of numbers, permutations, combinations, or factorials in your answers if you so choose.

The following is part of the city map of Anytown, USA.



(a) If one only travels east (i.e. to the right) or south (i.e. down), how many paths are there from A to C?

Solution:
This is the same as writing a string of E's and S's, where there are 8 E's and 6 S's. Each path is uniquely determined by such a string, and each path uniquely determines such a string. So of the 14 letters in such a string, we have to choose which 6 are S's (or equivalently which 8 are E's). Any of the following is a correct answer:

$$C(14,6) = C(14,8) = \binom{14}{6} = \binom{14}{8}.$$

(b) How many paths from A to C (again only traveling east or south) **avoid** passing through B?

Solution:
The number of paths that **do** pass through B is

$$\binom{7}{2} \cdot \binom{7}{4}$$

(because there are 7 blocks from A to B and 7 blocks from B to C, and we need to go 2 blocks south to get to B and 4 blocks south to get from B to C). So combining with the answer from (a), the number of paths that do not pass through B is the difference:

$$\binom{14}{6} - \binom{7}{2} \cdot \binom{7}{4}.$$

14. (10 pts.) For this problem you do not need to simplify your answers; you may leave permutations, combinations, and factorials (i.e. $P(n,k)$, $C(n,k)$, $\binom{n}{k}$, or $n!$) or products or sums of numbers, permutations, combinations, or factorials in your answers if you so choose.

A contingent of 8 Russians and 10 Americans meet for high-level talks.

(a) When the meeting starts, each Russian shakes hands with each American. How many handshakes are there?

Solution:
By the multiplication principle the answer is $8 \cdot 10 = 80$.

(b) During the break, each Russians slaps each other Russian on the back, and each American slaps each other American on the back, because they both think that the talks are going better for them than for the other guys. How many backslaps are there?

Solution:
This problem was actually phrased a bit ambiguously: if Bob slaps Bill and Bill slaps Bob, does that count as one backslap or two? The original idea was that this should be like handshakes or hugs, where it counts as one. But it would be reasonable to interpret it as two. So both answers were accepted.

Here is the solution if it counts as one. The number of Russian slaps is $\binom{8}{2} = 28$. The number of American slaps is $\binom{10}{2} = 45$. Thus the total number of slaps is $\binom{8}{2} + \binom{10}{2} = 73$.

If it counts as two, then you get double this, or $146 = P(8,2) + P(10,2)$.

(c) At the end of the meeting, each participant hugs each other participant, regardless of nationality. How many hugs are there?

Solution:
There are 18 people, so the total number of hugs is $\binom{18}{2} = 153$.

15. (10 pts.) For all parts of this problem, using factorials is preferred to actual numbers. Doug has 6 math books, 5 psychology books and 4 biology books.

(a) In how many ways can the 15 books be lined up on the shelf without regard to subject?

Solution:
$P(15, 15) = 15!$.

(b) In how many ways can the 15 books be lined up on the shelf if all books on the same subject have to be together? (Note that there is no specified order for the subjects; e.g. math-psychology-biology is different from psychology-biology-math.)

Solution:
There are 6! orderings just for the math books, 5! orderings for the psychology books and 4! orderings for the biology books. There are 3! orderings for the three different subjects. So the total number of possible orderings is

$$3! \cdot 6! \cdot 5! \cdot 4!.$$

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